

## Sensitivity study of frictional behavior by dimensional analysis in cold forging<sup>†</sup>

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### Abstract

In metal forming operations, material flow and quality of the product depend on the conditions at the billet/tools interface friction, lubrication, and surface finish. Of these parameters, friction is the most difficult to characterize and its influence is often difficult to predict because of its dependency on a variety of factors. Recently, through a number of investigations conducted, a linear relationship among shear friction factor, dimensionless load and tip distance was obtained. The aim of the present study is to see whether the linear relationship obtained in the previous works was fortuitous or genuine by applying a dimensional analysis introducing processing parameters such as contact pressure, ram velocity, viscosity of the lubricant, load, surface roughness and shear friction factor. From the dimensional analysis based on Buckingham  $\pi$  theorem and the data obtained from the tip test experiment, the present theoretical work derives dimensionless parameters and analysis of variance determines any correlation between the dimensionless parameters obtained. This work reconfirms the theoretical background of the previous experimental findings in the literature.

*Keywords:* Backward extrusion; Dimensional analysis; Forging; Friction

### 1. Introduction

The significance of friction considerations in bulk metal forming processes has been generally well recognized since friction affects the forming load/energy, tool life, metal flow, product quality, and manufacturing cost [1, 2]. The friction phenomenon is very complex since several parameters interact with one another and affect the forming process. Although a great deal of effort has been made in understanding the mechanism of friction, the subject of friction still needs more study.

Recently, a series of experimental investigations were conducted [3, 4] to unravel the frictional behavior at the contact between punch/material and material/die interfaces based on the backward extrusion of a cylindrical specimen of smaller diameter than the container. The radial tip distance generated from the die wall and the load were found to be linearly related to the shear friction factor different from the general phenomenon of nonlinearity of the friction involved in metal forming.

The aim of the present work is to establish a theoretical

background validating the previous experimental results and findings by deriving a set of non-dimensional parameters from physical variables involved with bulk metal forming processes. As a mathematical means, dimensional analysis will be used to transform the unknown relationship into a new function. The transformed data will then be connected by regression model to predict shear friction factor  $m_f$  in terms of the non-dimensional parameters. As a result, a simple linear model to predict shear friction factor will be obtained even though friction is known to be complex and highly nonlinear.

Constructing an empirical equation relating several parameters that affect friction experimentally can be enormous. To reduce these parameters in the most efficient form, dimensional analysis, which is a method for reducing complex physical problems to their simplest (most economical) forms prior to quantitative analysis or experimental investigation, will be used in the present work. In other words, dimensional analysis reduces a problem's degrees of freedom to the minimum and thus suggests the most economical scaling laws. This can be particularly useful in exploratory investigations of novel phenomena for which the equations and several parameters' interaction have not yet been fully articulated. In data analysis, there is the manipulation and transformation phase that is normally done to obtain approximate linear relationship. Dimensional analysis will be used in this study to replace this stage of data transformation. Dimensional analysis has this

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unique advantage that using the dimensionless variables constructed, any coefficients in a fitted model are also dimensionless and do not change if units of the measurement are changed. Further, any transformations are legitimate and the model remains dimensionally homogeneous. Although there are other methods of performing dimensional analysis, the method based on the Buckingham  $\pi$  theorem was employed in obtaining a solution in the present investigation.

**2. Dimensional analysis (DA)**

Buckingham [5] showed in his  $\pi$  theorem that if the original unknown relationship is represented by

$$f(x_1, x_2, \dots, x_n) = 0 \tag{1}$$

where the  $x_i$  are the variables, it can be transformed into a new function

$$\phi(\pi_1, \pi_2, \dots, \pi_{n-m}) = 0 \tag{2}$$

Here,  $n-m$  is the independent dimensionless products  $\pi_j$  of the original  $x_i$  variables. And  $m$  is the number of fundamental dimensions out of which the dimensions of the original physical variables are ultimately composed. These are mass (M), length (l), and time (t).

Assuming deformation at room temperature, the important variables in any metal forming operation that influence the friction will be normal pressure,  $p$ , ram speed,  $v$ , surface roughness,  $\ell$ , lubricant (viscosity),  $\eta$ , and the applied load,  $L$ . This can be stated as:

$$(m_f, p, v, \ell, \eta, L) = 0 \tag{3}$$

Since so many studies [6-8] have been published on dimensional analyses after the pioneering work of Buckingham, the detailed procedural derivation will not be repeated here.

Shear friction factor  $m_f$  is a non-dimensional number, and hence takes no part in the dimensional analysis itself but re-enters the procedure when we come to describe and fit the final model. By applying the Buckingham  $\pi$ -theorem [5] to these five physical factors with the three repeating variables  $L$ ,  $v$ , and  $\ell$ , Eq. (3) was transformed to give two dimensionless  $\pi$ -groups as:

$$\pi_1 = \frac{p\ell^2}{L} \quad \text{and} \quad \pi_2 = \frac{\eta\ell v}{L} \tag{4}$$

We re-introduce  $m_f$  to complete our function of  $\pi_j$  giving the dimensionally homogeneous equation as:

$$\phi\left(\frac{p\ell^2}{L}, \frac{\eta\ell v}{L}, m_f\right) = 0 \tag{5}$$

This was further simplified to give

$$m_f = \phi\left(\frac{\ell}{t}, \frac{L}{1000}\right) \tag{6}$$

where the characteristic load and length were selected as 1,000 kN and tip thickness  $t$ , respectively. Here,  $\ell$  was substituted by  $d$ .

DA cannot disclose the final form of the function or any of the numerical constants. Although shear friction factor was expressed as a function of tip distance and load as in Eq. (6), the form of this relationship can correctly be determined only by regression analysis. Having carried out a DA, we can then use all the power of traditional data analysis and regression to find and fit the unknown relationship, Eq. (3), using the new  $\pi_k$  variables. Also, we can now confidently carry out our data analysis (regression) in the knowledge that any function we find is guaranteed to be dimensionally homogeneous with the advantages described above. We now proceed to fit a regression model to predict the shear friction factor  $m_f$  from other physical variables.

**3. Experimentation**

The experimental data used were obtained from the tip test conducted at the laboratory by using a vertical hydraulic press with a maximum load of 400 metric tons as described in the literature [4].

In experiments, commercially available aluminum alloys Al2024-O, Al5083-O, Al6061-O, and Al7075-O, annealed carbon steel AISI 1010, and pure copper C12100 were used to examine the frictional behavior under various lubrication conditions. The lubricants used in experiments were grease, drawing oil, soap, MoS<sub>2</sub>, industrial oils with viscosity grades of VG32 and 100, and the household lubricant WD40. For specimens of Al2024-O and AISI 1010, phosphate-coated specimens were additionally used for tests.

Fig. 1 shows the experimental results in which a linear relationship was obtained between the load and tip distance for various experimental conditions.

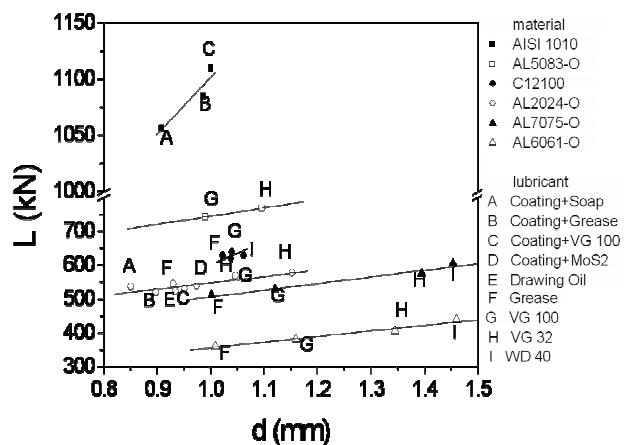


Fig. 1. A linear relationship between the load versus tip distance for various experimental conditions [4].

### 4. Results and discussion

The prediction models, relating  $m_{fp}$  the shear friction factor at the punch interface with two dimensionless parameters of load and tip distance, were reassessed by statistical regression study of the data obtained in the laboratory using the analysis of variance (ANOVA) as represented in Tables 1 and 2. All statistical analyses were performed by using Origin pro 8 software package.

For dependence of the shear friction factor on the dimensionless tip distance, the ANOVA indicated that the regression equation was statistically important as seen in Table 1. The size of the F value and the P value (Prob > F), respectively, showed further that the regression model is reliable and can explain almost 99.999% of the variation in the dependent variable,  $m_{fp}$ , due to the change in the independent variable,  $d/t$ . In other words, the dimensionless tip distance appeared to be a meaningful parameter and shear friction factor at the punch may sensitively vary depending on change of the tip distance.

In addition, the coefficients of the proposed regression equation are statistically different from zero. Thus, the shear friction factor can be successfully predicted by using the dimensionless tip distance. Also, adjusted  $R^2$  which measures the proportion of total variability explained by the model lends credence of 98.7% to the reliability of this regression equation.

This regression analysis gives us the linear fit between the dimensionless shear friction factor  $m_{fp}$  with respect to the dimensionless tip distance as shown in Fig. 2. In this figure, good agreement between the current model and laboratory measurements exists.

Table 1. Analysis of variance (ANOVA) of the plot of the shear friction factor at the punch versus the dimensionless tip distance.

	Degree of Freedom	Sum of Squares	Mean Square	F Value	P Value (Prob>F)
Model	1	1.0543	1.0543	1820.218	0.0001
Error	23	0.0133	5.792E-4		
Total	24	1.0676			

Intercept		Slope		Statistics
Value	Standard Error	Value	Standard Error	Adjusted R <sup>2</sup>
-0.958	0.0319	4.374	0.1025	0.987

Table 2. Analysis of variance (ANOVA) of the plot of the shear friction factor at the punch versus the dimensionless load.

	Degree of Freedom	Sum of Squares	Mean Square	F Value	P Value (Prob>F)
Model	1	0.0618	0.06183	1.4138	0.2466
Error	23	1.0058	0.04373		
Total	24	1.0676			

Intercept		Slope		Statistics
Value	Standard Error	Value	Standard Error	Adjusted R <sup>2</sup>
0.5451	0.1381	-0.254	0.2133	0.01695

Also, the prediction model relating  $m_{fp}$  with the dimensionless load was verified against the data obtained in the laboratory and the comparison is illustrated in Fig. 3. There is no good agreement between the regression model and the laboratory measurements in this figure.

For dependence of the shear friction factor on the dimensionless load, ANOVA indicated that the regression equation was not statistically important according to Table 2. Although the coefficients of the proposed regression equation are statistically different from zero, the values of many other parameters such as F value, P value (Prob > F), standard errors showed that the regression model was not significantly meaningful.

In addition, around 1.7% of adjusted  $R^2$  was rather too low at the same time. Therefore, this further casts doubt on the reliability of this model in predicting the shear friction factor. The model is incapable of full explanation of the variation in the dependent variable,  $m_{fp}$ , for any change in the independent variable,  $L/1000$ . It means that load variation is not a good parameter to predict the shear friction factor as represented in reference [9].

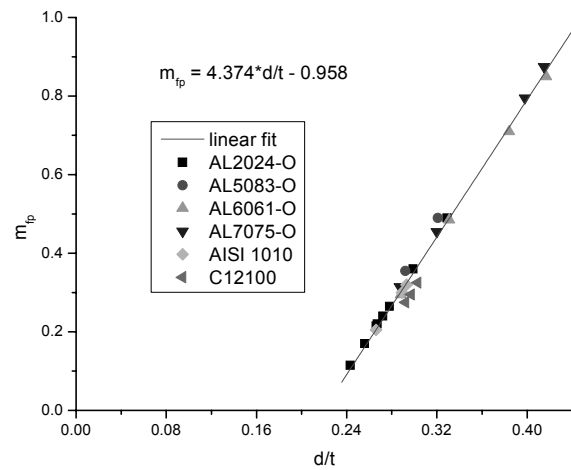


Fig. 2. Shear friction factor at the punch associated with the dimensionless tip distance.

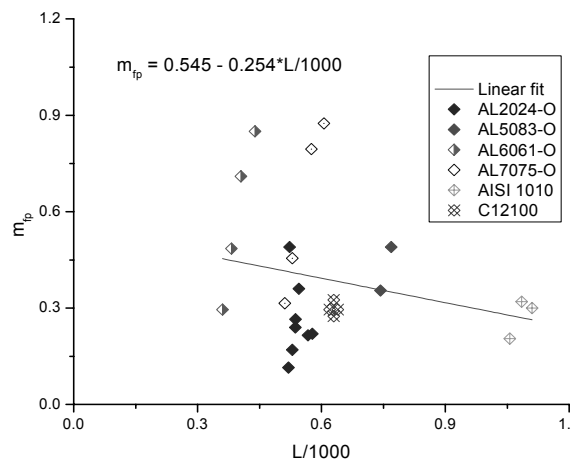


Fig. 3. Shear friction factor at the punch associated with the dimensionless load.

## 5. Conclusions

In this study, two  $\pi$ -groups  $d/t$  and  $L/1000$  kN were derived from five physical factors, such as normal pressure, ram speed, surface roughness, lubricant (viscosity), and the applied load by the dimensional analysis. To investigate dependence of the shear friction factor on  $d/t$  and  $L/1000$ , ANOVA was conducted and two regression models were compared. Based on the statistical results investigated, the model using  $d/t$  as the independent variable was found to be valid in successfully predicting the shear friction factor at the punch interface ( $m_{ip}$ ) than the other model using  $L/1000$  kN. Therefore, this study reconfirms that tip distance is a better parameter than load for predicting friction behavior. Finally, the present theoretical result has confirmed the veracity of linearity observed between the shear friction factor and dimensionless tip distance in the laboratory experiment.

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